

# GRAVITY SANS SINGULARITIES

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## Summary

Basis and limitations of singularity theorems for Gravity are examined. As singularity is a critical situation in course of time, study of time paths, in full generality of Equivalence principle, provides two mechanisms to prevent singularity. Resolution of singular Time translation generators into space of its orbits, and essential higher dimensions for Relativistic particle interactions has facets to resolve any real singularity problem. Conceptually, these varied viewpoints have a common denominator: arbitrariness in the definition of ‘energy’ intrinsic to the space of operation in each case, so as to render absence of singularity a tautology for self-consistency of the systems.

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1. Following powerful theorems in General Relativity (GR) gravitational singularities have acquired rare acceptability, unusual in physics. Singularity in a theoretical framework implies a critical situation in course of time. Relativity introduces complications of light cone (null paths, surfaces) and inseparability of Space from Time (causality, Hyperbolicity). Taking gravity as a central force, a null surface  $\vec{X}^2 - t^2 = 0$  defines for a spatial section the metric  $ds^2 = d\vec{X}^2 - dt^2 = R^2 d\Omega_2^2$ ; so, as a body contracts under gravity beyond  $r < R$ , a spherical 2-surface gets ‘trapped’. This forms basis of singularity theorems, beside sufficiency conditions for a minimum for proper time path functional  $\int d\tau$ , throughout. For not too specialised (relative to curvature components) velocity field  $v$ , this is better studied with the (Hamilton-Jacobi analogue) Raychaudhuri equation:  $\tau$ -rate trace of  $v$ -Lie derivative of the metric ( $2\dot{\theta}$ ) is quadratic in velocities with Ricci tensor coefficients (plus terms in  $-(\nabla v)^2$  and  $\nabla \dot{v}$ ); via Einstein equation, one obtains for ‘positive energy condition’, inevitability of singularity [1]. Despite, generality of this and related theorems, there are inherent limitations, as path or velocity field refers to an ideal test-particle: structureless, inviting no reaction, *e.g.*, in a closed system in mechanics, centre of mass may be fixed, but angular position can vary *ad libitum*; spin plays a crucial role here, and is ignored. Also ignored are forces (like Coriolis-magnetic field) that do no work - the only nongravity relativistic forces, giving added dimension to ‘interaction space’, where interaction energy ‘resides’, just as gravitational energy resides in space-time. Resolution of incomplete and non-global Hamiltonians in mechanics are also relevant.

2. Einstein [2] envisaged geometric Gravity as a generalization of Special Relativity (SR), in that: by weak principle of equivalence a test particle follows a geodesic, and by strong principle the metric is a covariant constant; both allow torsion. In

geometric terms, the first law in mechanics expresses vanishing of curvature and torsion; while a spherical rotator corresponds to vanishing curvature (not torsion) only; this yields [3] its symmetry group as  $SO(n+1)$  and spin-torsion relation.

A geodesic equation admits a further symmetry  $\Gamma_{\nu\sigma}^\mu \longrightarrow \Gamma_{\nu\sigma}^\mu + \delta_\nu^\mu \lambda_\sigma$ ; its antisymmetric part contributes to torsion to define precise spin-torsion relation in Einstein-Cartan theory [4]. The symmetric part redefines the path parameter. An integrable change  $\lambda_\sigma = \partial_\sigma \lambda$ , gives Einstein's [2]  $\lambda$ -transformation leaving curvature (with torsion) unchanged; Weyl's projective curvature is also unchanged, implying a same system of paths. Projective Einstein-Cartan structure is the most general, consistent with Equivalence principle and Einstein equations. The energy tensor gives only the contribution of matter, which balances or complements the gravitational field, as expressed in space-time geometry. Due to internal changes - no external energy is supplied - two sides of Einstein equations may, in principle, undergo spontaneous change governed by spin-torsion relation in Einstein-Cartan theory, or by projective change relating two (symmetric) connections to provide for singularity avoidance:

- (a) In a closed gravitating system a continual process of capture and expulsion is on, involving interconversion of orbital and spin angular momenta, and spin redistribution; it may be looked upon to generate torsion, leaving paths unchanged, but redefine Einstein and Energy tensors. In Raychaudhuri equation such a change is known [4] to provide for avoidance of singularity.
- (b) A spontaneous projective change, alters the (covariant) Ricci tensor additively [5] and may be used in Raychaudhuri equation to avoid singularity. Elsewhere we have treated Robertson-Walker metric for imploding cold body for singularity avoidance [6] and for Hawking radiation [5]. But the present argument is more general.

3. In mechanics for linear force the orbits are either only closed or open; the corresponding Schrödinger operators have respectively discrete or continuous spectra; here Hamiltonians generate finite canonical transformations and belong to  $sl(2; R)$  Lie algebra. For the Kepler problem, Hamiltonian vector field is known to be *incomplete* and non-global. By a change in time parameter, it is resolvable into 1-parameter subgroups of  $sl(2; R)$ , that have either discrete or continuous spectra. For Hamiltonians with more complicated spectra of uneven spread of point and continuous spectrum, one expects some representation of Virasoro algebra. For path-integral computations and uncertainty relations, only such globally Hamiltonian vector fields will do. *Thus incompleteness does not necessarily imply a singularity, when it can be resolved.* Direct application of this to GR singularities is perhaps problematic, as space and time are well entwined. But, there is a connection between approaches based on curves and 1-parameter groups via discrete subgroups of conformal group, implicit in Poincare's work on automorphic functions and bounded domains.

4. Relativistic Kinematics of particles describes all elastic/inelastic processes without a potential, as all interaction energy comes from the masses. For translation generators  $P_\mu$ ,  $mv_\mu \longrightarrow P_\mu - A_\mu$  defines the displacement field  $A_\mu$  as carrier of interaction energy modes, corresponding to forces (like Coriolis) which do no work. Taken as matrix valued - say, in basis of adjoint representation of a Lie algebra  $\mathfrak{g}$  of a compact group  $\mathcal{G}$  - it defines a vector space  $V^n$  added on to the space of tangents at each point of  $M^d$ , to give interaction space  $M^{d+n} = M^d \times \mathcal{G}$  locally. Since, for  $m \geq 0$ , the fields are defined over doublesheeted hyperboloid/cone, a particle may be conceived of as surrounded by fluctuating clouds of virtual particles and antiparticles, as well  $A_\mu$  modes in which they may unite to go back and forth. *We now*

*make a drastic assumption:* the entire energy  $m$  of a typical particle is determined by energy of a ‘mean’ Newtonian oscillator relative to proper time  $\tau$ ; *viz.*

$$\frac{1}{2}\mu\left(\frac{d\xi}{d\tau}\right)^2 + \frac{1}{2}\omega^2\xi^2 = \frac{1}{m}P_\mu P_\mu = m = m\frac{dt^2 - d\vec{X}^2}{d\tau^2}.$$

For time-fluctuation  $\xi = t$ , de Sitter space obtains as relativistic analogue of uniform acceleration. For spatial-fluctuation in  $N$ -dimensions, one obtains 5-dimensional anti-de Sitter (Ads) space  $\times S^{m-1}$ . Spatial part of Ads, in these coordinates, is a space of negative constant curvature, rewritten as

$$ds_4^2 = R^2 \left[ d\alpha^2 + (\sinh \alpha)^2 d\Omega_3^2 \right].$$

Analogue of Coulomb potential in such  $N$ -space is

$$\psi_N = \int (\text{cosech } \alpha)^{N-1} d\alpha \quad r=\alpha R \xrightarrow{R \rightarrow \infty} -\frac{mGR^{N-3}}{r^{N-2}}.$$

For  $N = 3$  it is  $(1 - \coth \alpha) \longrightarrow -MG/r$ ; the Schrödinger operator in space of negative curvature with this potential has finite number of eigenvalues; and as model of ‘black hole interior’ lowest eigenvalue gives measure of (BH entropy)<sup>2</sup> [7]. For  $N = 4$  for large  $R$ , potential is  $\lambda r^{-2}$ , for which the flat space Schrödinger operator has  $s$ -state eigenvalues [8]

$$-E_n \sim \exp[2B - (2n + 1)\pi(4\lambda - 1)],$$

where each real  $B \neq 0$  defines a new self-adjoint extension of Hamiltonian, with  $B \longrightarrow B + M\pi$  ( $n, M = \pm$  integers) leaving all unchanged. A particular disadvantage is that point spectrum extends from  $-\infty$  to 0. Consider now the bundle of all such extensions, and pick a subset where  $M$  and  $2n + 1$  adjust to give a finite lower bound (even degenerate) with proper orthogonality (and completeness, with continuous spectrum) preserved. *Thus particles sit here confined, as if free, but in a higher dimensional space with strong gravity;* and singularity is avoided. For the

case of the phase operator, bundle of all its extensions give [9] the  $kq$ -representation of Zak. Such interpretation is yet unavailable for  $B$ .

All four mechanisms cited (spontaneous spin/torsion or projective change; non-global, incomplete Hamiltonians; Relativistic interaction space), enabling avoidance of singularity, have a common denominator - arbitrariness in quantifying available interaction energy (*viz.* gravitational in GR, mechanics, and particle interactions) inherent in the definition of space in which the energy ‘resides’. In that sense it is a tautology and evidence of consistency of the systems considered.

*Acknowledgements:* This work forms part of a larger study of Origin and Structure of Time to appear in the e-print Archive. The ideas in Sec. 4 were originally floated at Boulder in 1970. Thanks are due to R. Jagannathan for invaluable help.

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